Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name: Differential Geometry

Subject Code : 5SC02MTC1 **Branch: M.Sc.(Mathematics)**

Semester : 2 **Date : 04/05/2016** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions.** **(07)**

a. State four Vertex theorems. **(01)**

b. Define: unit normal of $\bar{\gamma}$. **(01)**

c. Write the definition of curvature of $K(t)$ for unit speed curve. **(01)**

d. Give formula of arc length. **(01)**

e. The following statement is true or false? The surface $S = \{(x, y, z) \in R^3 : x^2 + y^2 + z^2 = 1\}$ can be covered by single patch. **(01)**

f. Define $f^*: T_pS_1 \times T_pS_1 \to R$ by $f^* < v, w >_p =$ ____________. **(01)**

g. Let $T: R^n \times R^n \to R$ be a symmetric bilinear map. If $T < x, x > = 0$ for all $x \in R^n$, then show that $T = 0$. **(01)**

Q-2 **Attempt all questions** **(14)**

a) If $\bar{\gamma}$ is a regular curve in R^3, then show that its curvature K is given by the formula **(07)**

$$K = \frac{\|\ddot{\bar{\gamma}} \times \dot{\bar{\gamma}}\|}{\|\dot{\bar{\gamma}}\|^3}.$$

b) Let $\bar{\gamma}$ be a regular curve in R^3 with nowhere vanishing curvature. Show that γ is spherical if and only if $\frac{\tau}{K} = \frac{d}{dt}\left(\frac{\dot{K}}{K^2\tau}\right)$ **(07)**

OR

Q-2 **Attempt all questions** **(14)**

a) State and prove Wirtinger's inequality. **(07)**

b) Let $\bar{\gamma}: (a, b) \rightarrow R^n$ be a parametrized curve. Show that $\bar{\gamma}$ is regular if and only if it has unit speed reparamerization. **(07)**

Q-3 **Attempt all questions** **(14)**

a) Let $\sigma: U \rightarrow R^3$ be a patch of a surface S containing a point p of S, i.e. $p = \sigma(u_0, v_0)$ for some $(u_0, v_0) \in U$. Then show that the tangent space to S at p is a vector subspace of R^3 spanned by the vectors $\sigma_u(u_0, v_0)$ and $\sigma_v(u_0, v_0)$. **(07)**

b) Find the first fundamental form of the surface $\sigma(u, v) = (\cos u \cos v\, , \sin v \cos u\, , \sin u)$. **(07)**

OR

Q-3 **Attempt all Questions** **(14)**

a) Let S_1 and S_2 be smooth surface, and $f: S_1 \rightarrow S_2$ be a local diffeomorphism. Show that f is local isometry if and only if for any surface patch σ of S_1, the patches σ and $f \circ \sigma$ of S_1 and S_2 respectively have the same first fundamental form. **(07)**

b) Compute the surface area of the sphere of radius R. **(07)**

SECTION – II

Q-4 **Attempt the Following questions.** **(07)**

a. Give an example of not oriented surface. **(01)**

b. Define: Gauss Map. **(01)**

c. Write Geodesic equation. **(01)**

d. Define: mean curvature. **(01)**

e. Derive $\sigma_{uu} \sigma_u = \frac{1}{2} E_u$ **(01)**

f. State Gauss Bonnet Theorem. **(01)**

g. $\Gamma_{11}^1 =$ _______________. **(01)**

Q-5 **Attempt all questions** **(14)**

a) Compute Gaussian curvature and mean curvature of the surface $z = f(x, y)$ where f is a smooth map. **(07)**

b) Let σ be a surface patch of an oriented surface S containing a point p of S, say, $p = \sigma(u, v)$. Then show that $II_p < w, x >= L\, dU(w)dU(x) + M(dU(w)dV(x) + dU(x)dV(w)) + NdV(w)dV(x)$ for all$w, x \in T_pS$, where $dU, dV: T_pS \rightarrow R$ are defined by $dU(w) = \lambda$ and $dV(w) = \mu$ where $w = \lambda\sigma_u + \mu\sigma_v \in T_pS$. **(07)**

OR

Q-5 **Attempt all Questions** **(14)**

a) Let $\sigma: U \rightarrow R^3$ be a regular surface patch and $(u_0, v_0) \in U$. For $\delta > 0$. Let $B_\delta = \{(u, v): (u - u_0)^2 + (v - v_0)^2 < \delta^2\}$. Let K be Gaussian curvature of the surface at p. Then $\lim_{\delta \to 0} \frac{A_{\bar{N}}(B_\delta)}{A_\sigma(B_\delta)} = |K|$. **(07)**

b) Let σ be a surface patch of an oriented surface S, and let p be in the image of σ; i.e. $p = \sigma(u, v)$, for some $(u, v) \in U$ then show that the matrix of W_p with **(07)**

respect to the basis $\{\sigma_u, \sigma_v\}$ of T_pS is $\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} L & M \\ M & N \end{pmatrix}$.

Q-6 **Attempt all questions** **(14)**

a) Compute the Christoffel's symbols of second kind for the circular cylinder $\sigma(u, v) = (\cos u , \sin u , v)$. **(07)**

b) Let $\bar{\gamma}(t) = \sigma(u(t), v(t))$ be a curve on a surface patch σ. Show that $\bar{\gamma}$ is Geodesic if and only if $\ddot{u} + (\Gamma_{11}^{1}\dot{u} + \Gamma_{12}^{1}\dot{v})\dot{u} + (\Gamma_{12}^{1}\dot{u} + \Gamma_{22}^{1}\dot{v})\dot{v} = 0$ and $\ddot{v} + (\Gamma_{11}^{2}\dot{u} + \Gamma_{12}^{2}\dot{v})\dot{u} + (\Gamma_{12}^{2}\dot{u} + \Gamma_{22}^{2}\dot{v})\dot{v} = 0$ **(07)**

OR

Q-6 **Attempt all Questions** **(14)**

a) Determine the Geodesics on a plane. **(07)**

b) Show that a unit speed curve on a surface S is Geodesic if and only if its geodesic curvature is zero every where. Also prove that any (part of a) line on a surface is a geodesic. **(07)**